Exam 1 Study Guide

I. Types of Equations and how to solve.

§2.1   A. 1st Order Linear — Standard form $y' + p(t)y = g(t)$

Method : Integrating Factor $\mu(t) = \exp\left(\int p(t)\,dt\right)$

§2.2   B. 1st Order Separable — $f(y)\,y' = g(x)$

Method : separate variables, $\int f(y)\,dy = \int g(x)\,dx$

§2.2   C. 1st Order Homogeneous — $y' = f(y/x)$

Method : ~~set~~ change of variables to homogeneous.

$$v = y/x \qquad v + x\frac{dv}{dx} = \frac{dy}{dx}$$

So, $\dfrac{1}{f(v) - v}\, v' = \dfrac{1}{x}$

Don't forget to back convert v to y/x.

§2.4   D. Bernoulli Equations — standard form $y' + p(t)y = g(t)y^n$

Method : rewrite as $\dfrac{1}{y^n}y' + p(t)\dfrac{1}{y^{n-1}} = g(t)$

Substitute ~~var~~ variables: $v = \dfrac{1}{y^{n-1}} \qquad \dfrac{dv}{dt} = \dfrac{1-n}{y^n}y'$

$\dfrac{1}{1-n}v' + p(t)v = g(t)$ is linear.

* Don't forget to back convert $v = \dfrac{1}{y^{n-1}}$

Exam I Study Guide

I. Types of Equations, (cont'd)

§2.5 E. Autonymous — $y' = f(y)$

Method: special case of separable.

§2.6 F. Exact, 1st order — $m(x,y) + n(x,y) y' = 0$

where $\frac{\partial}{\partial y} m(x,y) = \frac{\partial}{\partial x} n(x,y)$

Method. ① Find partial antiderivative for $m(x,y)$ wrt to x, i.e., Find $M(x,y) + h(y)$

so that $m(x,y) = \frac{\partial}{\partial x} \left[M(x,y) + h(y) \right]$

$h(y)$ is undetermined.

② Set $\frac{\partial}{\partial y} \left[M(x,y) + h(y) \right] = n(x,y)$

~~$\frac{\partial}{\partial y} M(x,y)$~~ or $h'(y) = -\frac{\partial}{\partial y} M(x,y) + n(x,y)$

③ Find antiderivative $H(y) + C$ for $h(y)$.

Solution is $\psi(x,y) = M(x,y) + H(y) = c$
(sigh!)

II. Other things you need to know how to do

§1.1   A. Sketch Direction fields and Integral Curves

§2.5   B. Find equilibria and phase lines for an autonymous equation.

§2.4   C. Apply Theorem 2.4.1 and 2.4.2 to find where a linear (respectively nonlinear) equation has valid solutions for a given IVP.

- Know when there are unique or non unique solutions.
- Know examples of where non uniqueness and "hidden" solutions exist.

§2.3   D. Lame story problems
~~Probably go~~ - Set up is most important
- Focus on problems of type 1-21 on §2.3.

§2.7   E. Euler's Method - Know how to do simple 3-4 step examples by hand.
- Know General Formula.

III. Types of Tasks You Are Expected to be Able to Complete.

    A. Solving any IVP cold.

    B. ~~Sett~~ Setting up a conversion of variables but not necessarily solving

    C. Determining Long term / Short behavior either by solving or using qualitative methods such as phase line or direction field.

    D. Finding where your solution is valid without necessarily solving the equation. Think §2.4.

    E. Anything else covered in class ~~or~~ or on HW.

I ~~can~~ emphasize that understanding how to get information about an equation without knowing any solution is just as important as finding a solution.

You may wish to work problems 1-32 on pp. 133-134 as good study/practice/review of techniques.